

Effective two-body interactions in the $s - d$ shell nuclei from sum rules equations in transfer reactions

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Average effective two-body interaction matrix elements in the s-d shell have been extracted, from data on experimentally measured isospin centroids, by combining the recently derived new sum rules equations for pick-up reactions with similar known equations for stripping reactions performed on general multishell target states. Using this combination of stripping and pick-up equations, the average effective matrix elements for the shells, $1d_{5/2}^2$, $2s_{1/2}^2$ and $1d_{3/2}^2$ respectively have been obtained. A new feature of the present work is that the restriction imposed in earlier works on target states, that it be populated only by active neutrons has now been abandoned.

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I. INTRODUCTION

Effective nucleon-nucleon interaction forms an important information input in any shell-model calculation of nuclear energy spectrum. However, the exact form of nuclear force being unknown, it is in itself quite a problem to obtain the matrix elements of effective two-body interaction. A common approach is to parametrise the interaction and then obtain the parameters by making fits to experimental data. Detailed calculations of energy spectra are required for this purpose which may involve the setting up and diagonalisation of large matrices.

An alternative approach is to obtain meaningful averages of effective interaction matrix elements in a simpler manner by making use of the energy-weighted sum rules [1,2,3]. These sum rules relate the energy centroids of residual nucleus states populated via a direct single particle transfer reaction to the average effective two-body interaction matrix elements. So far, sum rules for stripping reactions have been applied more widely than those for pick-up reactions. The complexity of the expression for D centroids, derived from monopole energy-weighted sum rule, increases as we move from the simplest case of inequivalent transfer [2], to the case of equivalent transfer. In a situation when the target state contains only neutrons in the transfer orbit, and an addition of a single particle to it is being studied, the expressions for isospin centroids are still fairly simple [3] and have been applied to a large number of cases [4,5]. When there is no restriction on the occupancy of the transfer orbit in the target state, the monopole sum rule for the stripping case [6] assumes more complexity by way of a two-body correlation term, $\langle H^{01} \rangle$, which defies analytical evaluation. For meaningful application of the sum rules, it is essential to eliminate this term in an appropriate manner.

Recently, explicit algebraic expressions have been reported [7] for isospin centroids of residual nucleus states obtained via single nucleon equivalent pick-up reactions from general multishell target states. These along with earlier similar expressions for centroids in the stripping case form a set which has been used [7] to obtain the $1f_{7/2}^2$ matrix elements after eliminating the ‘problem term’ $\langle H^{01} \rangle$.

The present article presents the results of a similar approach extending the application of these sum rules to stripping and pick-up reactions in the s-d shell nuclei as targets. For the sake of completeness, the expressions for isospin centroids for both stripping and pick-up cases are reproduced in the following section.

II. THEORETICAL OUTLINE

The isospin centroids, E_T^\pm (superscripts +,− indicate stripping and pick-up cases respectively) of residual nucleus states having isospin T, obtained via single particle stripping and pick-up reactions on a target state with

isospin T_0 are given [6,7] by

$$E_{T>}^+ - E^+(riz)$$

$$= \frac{\sum_k \{ \langle H_{ik}^{00} \rangle_{Tar} + \langle H_{ik}^{01} \rangle_{Tar} + (N_i - \delta_{ik}) q_{T>}^+(k) \overline{W_{ik}^{T=1}} + (N_i + \delta_{ik}) r_{T>}^+(k) \overline{W_{ik}^{T=0}} \},}{\langle \rho_i \text{ neutron holes} \rangle_{Tar}}, \quad (1)$$

$$E_{T<}^+ - E^+(riz)$$

$$= \frac{\sum_k \{ \langle H_{ik}^{00} \rangle_{Tar} - (\frac{T_0+1}{T_0}) \langle H_{ik}^{01} \rangle_{Tar} + (N_i - \delta_{ik}) q_{T<}^+(k) \overline{W_{ik}^{T=1}} + (N_i + \delta_{ik}) r_{T<}^+(k) \overline{W_{ik}^{T=0}} \},}{\langle \rho_i \text{ proton holes} \rangle_{Tar} + \frac{1}{2T_0} \{ \langle \rho_i \text{ proton holes} \rangle_{Tar} - \langle \rho_i \text{ neutron holes} \rangle_{Tar} \}}, \quad (2)$$

$$E_{T>}^- - E^-(riz) = \frac{\sum_k \{ \langle H_{ik}^{00} \rangle_{Tar} - \langle H_{ik}^{01} \rangle_{Tar} \},}{\langle \rho_i \text{ protons} \rangle_{Tar}}, \quad (3)$$

and

$$E_{T<}^- - E^-(riz)$$

$$= \frac{\sum_k \{ \langle H_{ik}^{00} \rangle_{Tar} + (\frac{T_0+1}{T_0}) \langle H_{ik}^{01} \rangle_{Tar} \},}{\langle \rho_i \text{ neutrons} \rangle_{Tar} + \frac{1}{2T_0} \{ \langle \rho_i \text{ neutrons} \rangle_{Tar} - \langle \rho_i \text{ protons} \rangle_{Tar} \}}, \quad (4)$$

In these equations, $T_> \equiv T_0 + \frac{1}{2}$; $T_< \equiv T_0 - \frac{1}{2}$; the summation index k runs over all the active orbits in the target state while i refers to the ρ_i ($\equiv j\frac{1}{2}$) orbit into (from) which the nucleon transfer occurs. Further

$$N_i = 2j_i + 1; \quad (5)$$

$$q_T^+(k) = \frac{3}{4} n_k + \frac{f(T)T_{0k}}{2T_0}; \quad (6)$$

$$r_T^+(k) = \frac{1}{4} n_k - \frac{f(T)T_{0k}}{2T_0}; \quad (7)$$

n_k = number of nucleons in the k th active orbit in the target state;

$$f(T) = T(T+1) - \frac{3}{4} T_0(T_0+1) \equiv \begin{cases} T_0 \text{ for } T_> \\ -(T_0+1) \text{ for } T_< \end{cases}; \quad (8)$$

T_{0k} = partial contribution of nucleons in the k th active orbit towards the target state isospin;

$E^\pm(riz) = E_0 \pm \epsilon_i$, with E_0 being the target state energy and ϵ_i , the single particle energy of transferred nucleon with respect to the chosen inert core.

$\overline{W_{ik}^{T=1}}$ and $\overline{W_{ik}^{T=0}}$ in equations (1) and (2) are $(2J+1)$ -weighted averages of two-body effective interaction matrix elements, W_{ikik}^{JT} , in isotriplet and isosinglet states, respectively, of one nucleon in the i th orbit and another in the k th orbit.

$$\langle H_{ik}^{00} \rangle_{Tar} = -\frac{1}{2}(1 + \delta_{ik}) E_{Tar}^{(2)}(i-k) \quad (9)$$

where $E_{Tar}^{(2)}(i-k)$ is the total two-body interaction energy of active nucleons in the i th orbit with those in the k th

orbit in the target state. $\langle H_{ik}^{01} \rangle_{Tar}$ is the isovector two-body correlation term given by
 $\langle H_{ik}^{01} \rangle_{Tar}$

$$= \frac{1}{2} \sum_{\gamma} \langle Targetstate | (2\gamma+1)^{1/2} W_{ikik}^{\gamma} [\{ (A^{\rho_k} \times A^{\rho_i})^{\gamma} \times B^{\rho_k} \}^{\rho_i} \times B^{\rho_i}]^{01} | Targetstate \rangle \quad (10)$$

where the symbols A^{ρ} , B^{ρ} etc. have their usual meanings [6,7]. This term has, so far, defied an analytical evaluation. But as can be seen from equations (1) through (4), this term can be eliminated by suitably combining any two of these.

Usually both $T_<$ states and $T_>$ states are not fully populated in a single particle transfer reaction, either because of experimental limitations or because one of these is, sometimes, theoretically untenable. In these circumstances, it is more advisable to look for one stripping and one pick-up reaction on the same target to get rid of this two-body correlation term.

It may, however, be mentioned that for target states having single nucleon occupancy and for those with larger nucleon occupancy but isospin zero, the isovector two-body correlation term, $\langle H^{01} \rangle_{Tar}$, identically vanishes.

III. CALCULATIONAL PROCEDURE

The set of experimental data which is of interest to us is a pair of one stripping and one pick-up reaction on the same target nucleus with stripping (pick-up) of a nucleon taking place into (from) a specifically chosen orbit. Depending on the isospins of residual nucleus states 'seen' via these experiments, two of the equations (1) through (4) are combined to eliminate the isovector two-body correlation term, $\langle H^{01} \rangle_{Tar}$. This procedure essentially provides us with one equation involving $\overline{W_{ik}^{T=1}}$ and $\overline{W_{ik}^{T=0}}$ as parameters (i referring to the particular transfer orbit and k running over all the active orbits in the target state).

For a given transfer orbit, belonging to a major shell, usually a large number of experiments are available in literature with different nuclei as targets so that the number of linear equations having the same set of parameters, $\overline{W_{ik}^T}$, exceeds the number of variables. The best values of the parameters are, therefore, found by making a least-squares -fit.

As far as the coefficients of $\overline{W_{ik}^{T=1}}$ and $\overline{W_{ik}^{T=0}}$ and the other quantities occurring in the linear equations resulting after the elimination of $\langle H^{01} \rangle_{Tar}$ are concerned, their evaluation is quite simple. The isospin centroids, E_T^{\pm} , are calculated from the experimentally measured energies and strengths (spectroscopic factors) of various states of the residual nucleus (having same isospin T). $E(riz)$, $E_{Tar}^{(2)}$ and the single particle energies of nucleons in different orbits with respect to the postulated inert core are calculated with the help of binding energy data. The values of quantities like n_k , T_{0k} etc. are based on the

assumption of a pure configuration for the target state, and the denominators on the right hand sides of the equations (1) through (4) are obtained from the well-known non-energy-weighted sum rules [8].

IV. RESULTS AND DISCUSSION

Although the equations (1) through (4) enable us to deal with any target state having a multishell configuration, we have presently restricted ourselves to target states having single active orbit. We believe that taking up the simpler applications as a first step would provide us with a better understanding of the underlying principles leading to the above equations. This would also help us in the understanding of the effect of enlarging the configuration space, on the two-body effective interactions when we subsequently study the cases of multishell target states.

The present work concentrates on the s-d shell region of the periodic table. Experimental data (energies, JT values and spectroscopic factors of residual nucleus states) were collected from literature [9-40] for equivalent transfer (both stripping and pick-up) of a nucleon involving the $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ orbits of various nuclei as targets. Table I gives a list of the reactions and experimental centroids used in the present analysis.

As mentioned earlier, we consider only one orbit to be active in the target state and the nucleon stripping (pick-up) taking place into (from) this very orbit. For transfer to $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ orbits, we treat ^{16}O , ^{28}Si and ^{32}S , respectively, as the inert cores. Following the procedure outlined in the previous section then provides us with three different sets of equations for the average interaction parameters, one set for $\overline{W}_{d_{5/2}d_{5/2}}^{T=0,1}$, second for

$\overline{W}_{s_{1/2}s_{1/2}}^{T=0,1}$ and the third for $\overline{W}_{d_{3/2}d_{3/2}}^{T=0,1}$. The best fitted values of the parameters obtained from these equations are presented in Table II. The root-mean-square deviations for the three sets of fitting done in the present calculation are 0.673, 0.267 and 0.196 MeV respectively. For comparison, the values of the interaction parameters calculated from the matrix elements obtained in some other works [41,42] are also given in the table.

As can be seen from table II, the presently calculated values of average interaction parameters are in close agreement with our previous results obtained using data from stripping reactions alone. There is also a reasonable agreement between our results and the average parameters obtained from the modified surface delta interaction (MSDI) matrix elements of Halbert et al. [42]. The average effective interaction parameters obtained by these workers using radial integral parameterization (RIP) and those obtained by Kuo and Brown [41] show large differences with our results. These differences could perhaps be partly due to larger vector space and configuration mixing used by these workers while we work in a trun-

cated space limited to one active orbit only and even in multishell target states allowing more than one active orbit, configuration mixing is not permissible.

We would like to point out that the basic merit of the present sum rules approach is its being analytical in nature. Instead of doing a mixed configuration calculation, trying to outdo a similar calculation done earlier, we are presenting an alternative simple approach to handle nuclear spectroscopy of the final nucleus involved in a single particle transfer reaction.

Equations, (1)-(4), of the manuscript, relating the isospin centroids to the effective interaction parameters, both in stripping and pick-up reactions constitute useful constraints for researchers working in the current area.

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Table I. List of the experiments from which the centroids have been obtained for the present study.

Transfer orbit	Target	Stripping Reaction [Reference]	Centroid		Pick-up Reaction [Reference]	Centroid	
			Isospin	Value (MeV)		Isospin	Value (MeV)
$1d_{5/2}$	^{17}O	$(^3\text{He},d)[9]$	$T_<$	2.214	—	—	—
		$(^3\text{He},d)[9]$	$T_>$	4.176	—	—	—
	^{18}O	$(^3\text{He},d)[10]$	$T_<$	2.436	—	—	—
		$(^3\text{He},d)[10]$	$T_>$	7.630	—	—	—
	^{19}F	$(d, p)[11]$	$T_>$	1.491	$(d, ^3\text{He})[12]$	$T_>$	3.220
		$(^3\text{He},d)[13]$	$T_>$	10.814	$(d, ^3\text{He})[12]$	$T_>$	3.220
	^{20}Ne	$(d, p)[14]$	$T_>$	1.190	—	—	—
		$(d, n)[15]$	$T_>$	1.410	—	—	—
	^{21}Ne	$(d, p)[16]$	$T_>$	3.520	$(p, d)[16]$	$T_<$	2.121
	^{22}Ne	$(d, n)[15]$	$T_>$	7.890	$(d, ^3\text{He})[12]$	$T_>$	0.000
$2s_{1/2}$		$(d, n)[15]$	$T_<$	1.870	$(d, ^3\text{He})[12]$	$T_>$	0.000
	^{23}Na	$(d, n)[17]$	$T_<$	2.204	$(^3\text{He}, \alpha)[18]$	$T_>$	2.916
		$(^3\text{He},d)[19]$	$T_>$	10.086	$(^3\text{He},\alpha)[18]$	$T_<$	2.532
	^{24}Mg	$(^3\text{He},d)[20]$	$T_>$	0.308	—	—	—
	^{25}Mg	$(^3\text{He},d)[21]$	$T_>$	1.208	$(d, ^3\text{He})[22]$	$T_>$	1.056
$1d_{3/2}$		$(^3\text{He},d)[21]$	$T_<$	0.664	$(d, ^3\text{He})[22]$	$T_>$	1.056
	^{26}Mg	$(^3\text{He},d)[23]$	$T_<$	0.000	$(d, ^3\text{He})[22]$	$T_>$	0.011
	^{29}Si	$(d, p)[24]$	$T_>$	1.484	—	—	—
		$(^3\text{He},d)[25]$	$T_<$	0.627	—	—	—
		$(^3\text{He},d)[25]$	$T_>$	2.314	—	—	—
30Si	$(^3\text{He},d)[26]$	$T_<$	0.000	$(p, d)[27]$	$T_<$	0.000	—
	^{31}P	$(^3\text{He},d)[28]$	$T_<$	1.151	$(^3\text{He},\alpha)[29]$	$T_<$	0.600
^{33}S	$(d, p)[30]$	$T_>$	2.480	—	—	—	—
	$(^3\text{He},d)[31]$	$T_<$	0.289	—	—	—	—
^{34}S	$(d, p)[32]$	$T_>$	0.000	$(^3\text{He},\alpha)[33]$	$T_<$	0.259	—
	$(^3\text{He},d)[34]$	$T_<$	0.000	$(^3\text{He},\alpha)[33]$	$T_<$	0.259	—
^{35}Cl	$(d, p)[35]$	$T_>$	0.862	$(d, ^3\text{He})[36]$	$T_>$	0.000	—
	$(^3\text{He},d)[37]$	$T_<$	1.945	$(d, ^3\text{He})[36]$	$T_>$	0.000	—
^{36}Ar	$(d, p)[38]$	$T_>$	0.000	—	—	—	—
	$(d, n)[39]$	$T_>$	0.000	—	—	—	—
^{37}Cl	$(^3\text{He},d)[37]$	$T_<$	1.915	$(p, d)[40]$	$T_>$	4.299	—

Table II. Average two-body interaction parameters (in MeV).

	$W_{d_{5/2}d_{5/2}}^{T=1}$	$W_{d_{5/2}d_{5/2}}^{T=0}$	$W_{s_{1/2}s_{1/2}}^{T=1}$	$W_{s_{1/2}s_{1/2}}^{T=0}$	$W_{d_{3/2}d_{3/2}}^{T=1}$	$W_{d_{3/2}d_{3/2}}^{T=0}$
Present calc.	-0.636	-2.80	-0.559	-2.30	-0.326	-2.16
Previous calc. ^a	—	—	-0.537	-2.22	-0.356	-2.22
Kuo and Brown ^b	-0.398	-2.31	-2.21	-3.54	-0.127	-1.68
Hamiltonian ‘RIP’ ^c	-0.171	-2.32	-1.72	-4.32	-0.035	-2.38
Hamiltonian ‘MSDI’ ^c	-0.248	-3.50	-0.584	-3.28	-0.266	-3.43

^aReference [5].

^bReference [41].

^cReference [42].

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